Jens Høyrup

# A NEW EDITION OF THE *METRICA* OF HERON OF ALEXANDRIA

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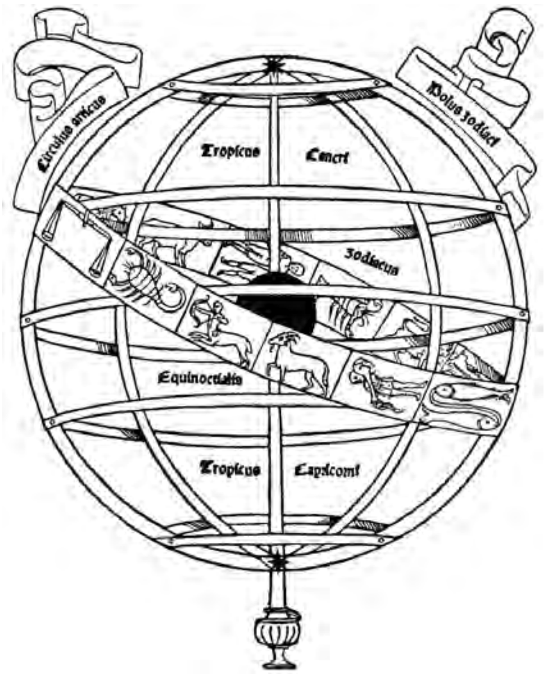


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# SOMMARIO

ESSAY REVIEWS

# A NEW EDITION OF THE *METRICA* OF HERON OF ALEXANDRIA *

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Quoting a letter to the Emperor from February 1865,[1] Emmanuel Miller announced in 1868[2] to have been shown in the library of the Old Serail of Constantinople "a very beautiful manuscript from the 11th century, containing the writings of Heron of Alexandria," which it "would be very important to compare with an edition of this famous mathematician, and in particular with the excellent memoir of M. Henri Martin, of Rennes, about the writers that have carried the name of Heron."[3]

The Old Serail library was not easily accessible – visit to the Serail was granted to foreign ambassadors taking their leave and normally to nobody else.[4] A few years after Miller's announcement, however, Anton Dethir, director of the Imperial Ottoman Museum, included a (fairly complete) list of the titles contained in the same manuscript in an inventory of such manuscripts as had already seen by foreign scholars – ascribing it now to the 12th century.[5] Then, in 1887, Friedrich Blass was admitted, and in the next year[6] he first said about the manuscript in

* Essay review of Fabio Acerbi, Bernard Vitrac (eds., trans.), *Héron d'Alexandrie, Metrica*, Pisa-Roma, Fabrizio Serra, 2014.

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1 Published according to Miller (p. I, n. 1) in *Le Moniteur* (probably *Le Moniteur Universel*), 13 March 1865.

2 Miller, 1868, p. v.

3 That is, Martin, 1854. My translation into English, as everywhere in the following where nothing else is stated.

4 Abel, 1878, p. 564.

5 *Ibid.*, p. 565.

6 Blass, 1888, pp. 220, 222.

question that it contains "The Geometry of Euclid – Heron of Alexandria on Measures," dating it again to the 12th century; next he quoted Dethir's more complete description via Abel. Only Richard Schöne was competent and interested enough in such matters to discover in 1896 that the manuscript contains Heron's *Metrica*, so far only known from a reference in Eutocios and from a fragment which Paul Tannery had identified in 1894.[7] In 1903, finally, Hermann Schöne (son of Richard) published the first critical edition (with a German translation) of what turned out to be the only mathematical work certainly written by Heron (excepting some aspects of the *Dioptra*);[8] he returned the date of the manuscript to the 11th century. In the same volume he also published the *Dioptra*.

These editions constitute the third volume of Heron's *Opera quae supersunt omnia*. The remaining mathematical treatises of the Serail manuscript (which I shall henceforth designate **S**, following a convention going back to J.L. Heiberg)[9] were published as volumes IV and V of the same series (similarly with German translation) together with most of those that had been published by Friedrich Hultsch in 1864.[10] Vol. IV contains the *Definitiones*,[11] and vol. V *De mensuris*.[12] Most of what remains is collected under the headings *Geometrica* and *Stereometrica;* both names go back to Hultsch, but Heiberg combines what comes from the manuscripts used by Hultsch (mainly **A**, written in 1183, and **C**, from c. 1300) with those parts of **S** that do not belong to the *Metrica*, and with supplements belonging to a Vatican manuscript labelled **V**. Where the two manuscript groups **A+C** and **S+V** deal with approximately the same subject-matter, their contributions are arranged in parallel columns;[13] matters dealt with only in **S+V** are inserted into the text where Heiberg found it fitting.[14] The outcome should not be taken as reconstructions of works from Heron's hand – Heiberg is emphatic about that. In the preface to vol. IV[15] he says to have found it adequate (because of the partial thematic overlap) "to pile up, so to speak, everything geometric and everything stereometric into two huge bulks" (*omnia geometrica et omnia stereometrica in duas quasi moles congerere*); the *prolegomena* to vol. V (p. XXI) repeat this warning, and insist that the material contained in the *Geometrica* does not come from Heron's hand (*ab Herone profecta non est* – p. XXI), and that the same holds for the *Stereomet-*

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7 TITTEL, 1912, col. 1013.

8 SCHÖNE, 1903.

9 HEIBERG, 1912a, p. XII.

10 HULTSCH, 1864.

11 HEIBERG, 1912a.

12 HEIBERG, 1914.

13 The objection of Fabio Acerbi and Bernard Vitrac (henceforth A&V) on p. 442 and elsewhere that the parallels are *forcés* is thus beside the point.

14 Heiberg had taken over the task after the death of Wilhelm Schmidt in 1905, as explained in HEIBERG, 1912a, p. III. Schmidt was responsible for much of the copying of manuscripts, but the editorial decisions appear to have been Heiberg's own.

15 HEIBERG, 1912a, p. III.

*rica* (p. xxix). What Heiberg has published are thus presented as two collections of "matters geometric" and "matters stereometric" which should not be understood to have any global coherence, and whose relation to Heron is at best highly dubious. Over the years, unfortunately, these cautious statements, formulated in Latin in introductions, have been overlooked by a number of colleagues jumping directly from the title pages to the texts.

No mistake of this kind was involved when Otto Neugebauer took "the geometrical writings of Heron, whether authentic or merely ascribed to him" [16] to represent the second of two

> separate types of "Greek" mathematics. One is represented by the strictly logical approach of Euclid, Archimedes, Apollonius, etc.; the other group is only a part of general Hellenistic mathematics, the roots of which lie in the Babylonian and Egyptian procedures,[17]

to which may be joined a quotation from a slightly later publication from Neugebauer's hand,[18] namely that

> one can no longer doubt that the discoveries of the Old Babylonian period had long since become common mathematical knowledge all over the ancient Near East. The whole tradition of mathematical works under the authorship of Heron (first century A.D.), Diophantus (date unknown), down to the beginning Islamic science (al-Kwârazmî, ninth century) is part of the same stream which has its ultimate sources in Babylonia.

This became the general tenor of the reception of Heron and the pseudo-Heronic corpus; in spite of the well-known animosity of Evert M. Bruins against Neugebauer he agreed with him on this point, and even gives the discovery of cuneiform mathematics as a main reason that a new edition of the *Metrica* was needed.[19] He also points out that

> the desire to give as many texts as possible often resulted in a compilation of the material from different codices which can be compared to editing one book in which all the different revised editions of different books of the same author on the same subjects have been interlaced.

Therefore, in 1964 he published a complete edition of **S** in three volumes – one containing a facsimile, one a Greek transcription, one an English translation and a commentary.

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16 Neugebauer, 1957, p. 80. Elsewhere in the book (pp. 47, 52), indeed, Neugebauer quotes the *Metrica* precisely and correctly, while p. 146, also correctly, refers to "writings which go under the name of Heron of Alexandria" but are not his.

17 Neugebauer adds the caveat that "Of course, since the Hellenistic period, even the writings of Heron and related documents show the influence of scientific Greek geometry."

18 Namely Neugebauer, 1963, p. 530.

19 Bruins, 1964, I, p. ix.

Now, after another fifty years, A&V have published a new edition of the *Metrica*, which is actually much more than a mere edition of that work – the reason for the preceding protracted introductory exercise, which will serve in what follows. Following Giancarlo Prato (whose work I have not seen, and whose arguments are not reported), A&V identify the copyist of **S** as Ephrem, also known for a good copy of the *Elements*, and accordingly date the manuscript itself to c. 960 (p. 92).

As it turns out, getting a digitized copy of the manuscript is as difficult today as it was getting access to the manuscript itself in the 19th century; in consequence the edition is based on Bruins' facsimile.

Apart from that, A&V have no kind words for their predecessors. What Schöne had produced is characterized on p. 97 gratuitously as "a bad edition" (*une mauvaise édition*), neither argument nor specification being offered. As concerns Bruins, they go on, "one cannot even speak of an 'edition' of the text and the scholia;" in this case at least a sham argument is given, namely a quotation from Bruins concerning his *translation* principles (thus not the edition at all) supposed to reveal his total incompetence.[20] On p. 96 he is further quoted for the opinion that the "copyist clearly did not understand what he was writing," for which Bruins indeed gives the

> striking example [that] on fol. 77 he copied for the area of the equilateral triangle 43 $^1/_3$ $^1/_{38}$ $^1/_{40}$ $^1/_{41}$, splitting up the indication of the next section λημμα into the fractions $\overline{\lambda\eta}$ $\overline{\mu}$ $\overline{\mu\alpha}$![21]

To this A&V object (p. 96, n. 124) that

> the writing $\overline{\lambda\eta}$ $\overline{\mu}$ $\overline{\mu\alpha}$ [...] cannot indicate the fraction $^1/_{38}$ $^1/_{40}$ $^1/_{41}$ as Bruins pretends, but at most the numbers 38 40 41, since none of the former [i.e., the unit fractions] are marked by a horizontal stroke.

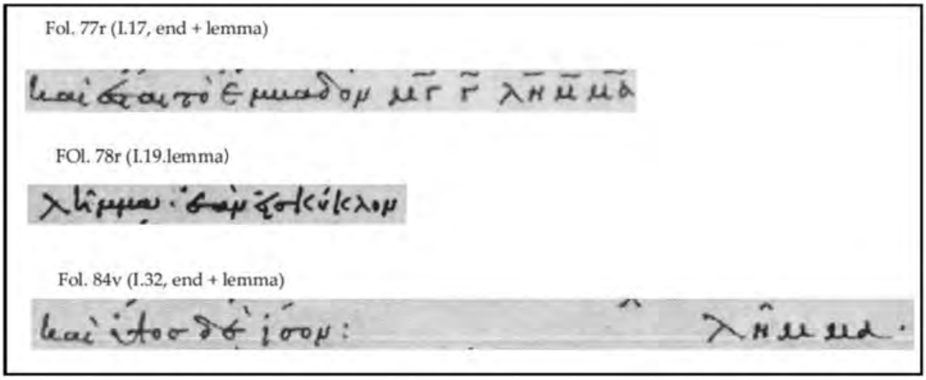
Whoever inspects the facsimile will see that the preceding $^1/_3$ is written $\overline{\gamma}$, that is, exactly "marked by a horizontal stroke"[22] – see the figure. In any case, whether the copyist meant one or the other thing (or nothing at all), it is obvious that the word λημμα has been transformed into a sequence of numbers or fractions, which is essentially Bruins's point.[23] Why this should be contradicted by Heiberg's

20 *Ibid.*, I, p. x. According to my own experience with Bruins, he certainly made mistakes. But the reason was not incompetence (he sometimes had very good ideas) but an extreme stubbornness; once he had conceived an idea, it would take months and many letters to make him understand an underlying blunder (even when what was to be proved was that he had been right himself 25 years ago). When he finally understood, he would regularly switch position and impute the error on the discussion partner.

21 *Ibid.*

22 Obviously, we all make mistakes, but that this should be one strains credulity. If against all odds it *is* a mere mistake, one should perhaps question the reliability of the edition as a whole. The reviewer, having no training in Greek palaeography, shall not persevere.

23 All three modern editions make the reverse correction and restored λημμα. Since the




observation (quoted with approval as if it were an objection to Bruins) that the manuscript is written in a "beautiful and skilful hand" (*pulchra peritaque manu*) is an enigma.[24]

None the less, one can still hope the present work to be better than the predecessors, and not only because it draws (as also said on p. 97) on Schöne's work as well as on the proposals of those who had offered him advice or commented philologically upon his edition.[25] It also contains much more.

A general introduction of 125 pages starts by discussing what we know and what we do not know about Heron's date, about his works, and about the general character of his work. It goes on with an analysis of the *Metrica* – its contents, its lexicon, its operatory terminology, and its numerical notations – and with a description of the manuscript and of the principles governing the edition and the translation. This is followed by a number of appendices pertinent to the preceding arguments. The first of these – the text of *Dioptra* 35 with translation and commentary – has to do with the determination of Heron's epoch, and the second with Heron's format for cross-references etc. Two regard Heron's methods for extracting square and cube roots, in the *Metrica* and as reported by Theon in the *Prolegomena to the Almagest;* one presents *Dioptra* chapters 28-29 – theorems concerning the division of areas (here only the Greek texts are given, as rather often).

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copyist did not do so, he cannot have thought about what he was writing, irrespective of whether his original was mistaken or not. As can be seen in the figure, the word λημμα here, beyond being split up and carrying the superscript strokes, follows immediately after the preceding numerical result without punctuation. The two other occurrences of the word are written either on a new line (fol. 78$^{r}$) or after punctuation and a very large break (fol. 84$^{v}$); both also begin with a somewhat enlarged λ.

24 As A&V observe on p. 86, the copyist meticulously renders lacunae or corrupt passages in his original by blank spaces. This does not exclude that he understood the mathematics of the text but at least demonstrates that he did not use such understanding as a basis for repairs.

25 Bruins is not mentioned here, and in fact only his geometric reconstruction of the taking of a cube root is drawn explicitly upon on p. 124.

A third appendix compares the ways "Heron's formula" for the area of a triangle is presented in *Dioptra* 30 and *Metrica* I.8; two, finally, inform about some details characterizing manuscript **S**.

The edition itself, with facing French translation, apparatus and notes, takes up 216 pages. In comparison, Schöne's edition with translation and more modest apparatus[26] takes up 184 much smaller pages, and that of Bruins (edition alone) 54 pages. This reflects the choice of A&V to let all text chapters begin at a new page, and to let diagrams follow after the text chapter where they belong (not quite as done in the manuscript, whose copyist could hardly afford a similar waste of parchment); the aesthetic impression is pleasant. The diagrams themselves respect the principle of conformality; since they are well made in the manuscript, this is meaningful. It is certainly somewhat disturbing for a modern reader that angles that are stated to be right are sometimes conspicuously oblique (etc.), but it is healthy for understanding the original thinking about diagrams. Schöne, or rather his typesetter, inserts the diagrams in indentions, in the manner of the manuscript (not necessarily towards the end of the corresponding text, as does the manuscript in most cases, but as it fits the page); he also draws them conformally, but in many cases he omits numerical values written into otherwise lettered diagrams.[27]

The translation also aims at being conformal, at least as regards the frozen formulaic expressions characterizing Greek mathematical style and its use of logical connectors and other particles (p. 98). It is attentive to the gender of the definite article in elliptic phrases, since this reveals which noun is omitted but presupposed (for instance, showing whether simply the number "17" or "17 units" is meant).[28] It does not promise to treat the terminology for operations and the tense and aspect of verbs consistently, but as far as I have observed it does so; the choices can be read out of section 5 of the general introduction, "La terminologie opératoire dans les *Metrica*" (pp. 74-81). Only one choice seems objectionable to the reviewer: the translation of μίαν as "au moins une" (pp. 181, 189, explained in note 121) where one and no more is asked for mathematically and by Heron.

The edition is followed by three complementary studies. The first discusses "analytic procedures in the Heronian geometric writings." In the end of *Metrica* I.6, Heron announces indeed that he is going to change the style of his exposition; what has so far been found by calculation (referred to until I.5 and even a few lines earlier in I.6 as μέθοδος, in this connection "procedure" rather than

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26 Unlike A&F, after all, Schöne had no earlier readings with which to compare.

27 Bruins omits all the diagrams but one (illustrating III.8) in the edition and translation, and this one is rendered in true proportions, not in those of the manuscript; but all diagrams can be seen in the facsimile.

28 Formulaic expressions and the character of elliptic phrases are analyzed in Aujac, 1984 and Netz, 1999, pp. 127-167. None of these publications are ever mentioned by A&V, not are their authors.

"method"), will henceforth be produced by analysis through "synthesis of numbers." And indeed, in what follows this use of μέθοδος disappears (with a sole exception in III.3), and Heron tries to display devotion to the method of analysis and synthesis.[29] In the analysis parts of his demonstrations (which *are* indeed the demonstrations), he often proceeds in steps "since *this* is given, even *that* will be given." Sometimes these correspond directly to propositions of Euclid's *Data*, sometimes they only follow from combination of a whole "chain" of such operations.[30] The first complementary study accordingly investigates the use of such knowledge in the *Metrica* and in Heron's commentary to *Elements* II.2–10 (as known through al-Nayrīzī), finding "chains" in the former but not in the latter case. The absence of chains from the proofs of the commentary is explained from the purpose of that work; since it thus does not illuminate the *Metrica*, one may ask why the commentary is at all discussed. An appendix to the study (37 of its 47 pages) lists the linguistic formulae determining the format of all propositions that indicate a procedure.

The second complementary study (17 pages) describes "the stylistic codes of ancient Greek mathematics and their manifestations"[31] in the *Metrica* – first in the demonstrative parts, next in the "algorithmic procedures" (a notion to which we shall return), where however only the stylistic codes in the *Metrica* are dealt with, not their relation to those of Euclidean-Archimedean mathematics nor their links to the codes of other works (Greek or otherwise) belonging to Neugebauer's second type.

The third complementary study, by far the most weighty and also the longest (160 pages), deals with "The afterlife of the *Metrica*. The Greek metrological corpus" (that is, roughly speaking, the writings contained in volumes IV and V of the *Opera quae supersunt omnia*[32] – but initially A&V point out that using these *bulks* (Heiberg's *moles*) as if they were *works* would be misleading, for which reason they uncouple the constituent parts in their discussion. This is reasonable and helpfull: the user of the texts as contained in the *Opera* will only be able to do as much by

[29] One may have doubts whether a numerical rule, even if mapping more or less precisely a preceding analysis, can really be considered a synthesis, and A&V occasionally express such doubts (e.g., "more or less like a geometric synthesis" – p. 365); but justifiable or lip-service, this is what Heron does.

[30] To make things clear: chains as such are not in the text, not even hinted at. A&V use the term when "several theorems from Euclid's *Data* must be mobilized in order to reach the conclusion" (p. 167 n. 74, commenting upon I.8); they are reconstructions needed if one wants Heron's argument to build on the *Data*. By using the concept A&V seem to intimate that Heron uses the *Data*, but they do not claim this directly. As far as the reviewer can see, the need to introduce the "chains" might rather suggest Heron to use what could be considered general knowledge and not the Euclidean treatise – for example (*Metrica* II.6, see p. 265 n. 47), that a triangle is given if all three sides are given.

[31] *Avatars* – maybe the word used by A&V is inspired by contemporary pop/computing culture and thus to be translated instead as "stand-ins."

[32] Heiberg, 1912a, 1914.

making intensive use of Heiberg's descriptions of the manuscripts in the introductions – the reviewer speaks from proper experience.[33]

Before describing the make-up of the corpus, A&V give a sketch of its editorial history from the 16th century up to Heiberg, and a "summary inventory" of the main manuscripts. Then follows analysis, first of the main constituents of the corpus, then of minor components.

All of this serves as background for more properly historical inquiries:

– the questions whether the pseudo-Heronic writings are derived from the *Metrica*;
– the problems of chronology and attribution;
– the relation of Heron and the pseudo-Heronic material to the Roman agrimensors;
– the question how much of the corpus reached the Islamic Middle Ages, and the various appearances of "Heron's formula" for the triangular area in the Islamic and Latin Middle ages.

Eight pages about the familiarity of the Byzantine Jewish scholar Mordekhai Comtino (1402-1482) with the Heronic as well as the pseudo-Heronic parts of **S** closes the study proper. However, once more there are a number of appendices. One lists "the so-called school papyri of geodesic character," others describe the contents of the pseudo-Heronic corpus or confronts (select aspects of) it with the *Metrica*.

In the end come, beyond the bibliographic references, a number of indexes:

– of Greek words and verbal forms;
– of names;
– of geographical locations;
– of manuscripts and papyri;
– and of technical terms and notions.

In particular the *index graecitatis* (29 pages, listing all grammatical forms that occur in the *Metrica* edition with page and line)[34] and that of terms will be useful. In my scattered checks, all index references were correct, and no occurrences were omitted.

So, the volume under review will be an important tool for everybody working on or in the vicinity of Heron's mathematics – though certainly, as regards

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33 That A&V disregard Heiberg's prefaces and only complain (p. 432) that he has "produced two philological 'monsters' which the contents of the manuscripts now known by him [namely, after the discovery of **S**] did not impose," on the other hand, suggests *mauvaise foi*.

34 Even the definite articles – which, as we know at least since (NETZ, 1999, pp. 92, 96 and *passim*) are important because the nouns they refer to are often omitted but in the context determined unambiguously by the gender of the article; cf. above, note 28. Schöne's edition contains a similar index, which I have not checked for completeness. Bruins offers none.

the "vicinity," only if one has the editions of the pseudo-Heronic writings at hand.[35]

However, because of a number of problematic features of A&V's work, it should be used with care. In agreement with the pseudo-Aristotelian principle *amicus Plato, sed magis amica veritas*, they have to be discussed.

Perhaps not really problematic but still somewhat objectionable is the absence of translations of a number of Greek terms and passages; this omission implies that the reader who is not well versed in Greek will have to take many of the conclusions derived by A&V on faith, being unable to judge this aspect of the argument.

Next, the book refers throughout to a notion of "algorithms". As far as I am aware, the description of early mathematics in terms of algorithms goes back to an article by Donald Knuth entitled "Ancient Babylonian Algorithms."[36] As Knuth saw things (p. 622), the

> Babylonian mathematicians […] were adept at solving many types of algebraic equations. But they did not have an algebraic notation that is quite as transparent as ours; they represented each formula by a step-by-step list of rules for its evaluation, i.e. by an algorithm for computing that formula. In effect, they worked with a "machine language" representation of formulas instead of a symbolic language.

This was an adequate description of the *translations* of the time, but as it has turned out since then not of what goes on in the texts, which actually describe manipulations of geometric configurations involving measured entities. Be that as it may, in the present context it is more interesting to see what characterizes the texts (as Knuth understood them) as *algorithms:* namely to be "step-by-step list[s] of rules" for evaluating supposedly underlying "formulas."[37] What Knuth found was of course not *rules* but the actual numerical calculations of paradigmatic examples; but as indeed intended by the Babylonian writers, he understood these steps as representing general procedures – quite in agreement with the charac-

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[35] Fortunately, at least for the time being they are accessible on the web. Links to all volumes of the *Opera* will in the moment of writing (20 February 2015 – and still 8 April 2018) be found on http://www.wilbourhall.org/index.html#hero (a sub-page of a site in memory and honour of David Pingree). Also for the time being, volumes 3, 4 and 5 can be downloaded from: http://gallica.bnf.fr/ark:/12148/bpt6k251883.r=.langEN; http://gallica.bnf.fr/ark:/12148/bpt6k25553j.r=.langEN; http://gallica.bnf.fr/ark:/12148/bpt6k25160q.r=.langEN.

[36] Knuth, 1972.

[37] This is in full agreement with the explanation of the concept in a recent textbook and thus with what the word means to those who use it professionally (Cormen, 2009, p. 5): "Informally, an *algorithm* is any well-defined computational procedure that takes some value, or set of values, as *input* and produces some value, or set of values, as *output*. An algorithm is thus a sequence of computational steps that transform the input into the output. We can also view an algorithm as a tool for solving a well-specified *computational problem*. The statement of the problem specifies in general terms the desired input/output relationship. The algorithm describes a specific computational procedure for achieving that input/output relationship".

terization of such calculations as *epēšum* ("doing" / "making"), as μέθοδος in the beginning of the *Metrica* and in the pseudo-Heronic writings, and as *regula / regola* in medieval Latin and Italian writings of the same kind.

Knuth complains on p. 674 that he only finds

> straight-line calculations, without any branching or decision-making involved. In order to construct algorithms that are really non-trivial from a computer-scientist's point of view, we need to have some operations that affect the flow of control.

This linearity, of course, corresponds exactly to the Babylonian, Greek and Medieval "rules." It is very difficult to find anything like the "if" or "until / while" prescriptions of modern algorithms – the most obvious exception being certain medieval explanations of the double false position, which state what to do if the required result falls between and what to do if it falls outside the interval determined by the two guesses.

So, why speak of "algorithms" instead of "rules" or "procedures," as historians of mathematics would do until a few decades ago? The reason is probably that culture of euphemism that goes under the name "political correctness" (itself a euphemism). "Rules" were sometimes (though hardly by those who were really acquainted with the material in question, which is often too advanced for that) "assimilated to a scarcely rational empirical approach" (thus A&V, p. 31),[38] whereas "algorithms" have a fancy ring of being modern mathematics. The easy objection is that even modern users of algorithms (not least the algorithms embedded in their computers and smartphones) almost without exception take them as they are, without understanding how or why they work.

The most obvious advantage of the algorithmic terminology in the analysis of early mathematics is probably *funding:* it makes applications sound up-to-date – which research council would give money to a project about "rules" or "procedures?" The book under review is indeed the outcome of work under a grant and project "ANR ALGO [ANR– –09– –BLAN– – 0300– –01]," as stated on p. 11.

In most of history, scholarship (when not, as rarely, the hobby of wealthy amateurs) was conditioned by the possibilities of patronage. That is reflected in dedications beyond number but rarely in the works themselves. But times are changing – they always are – and for scholars the present times are perhaps less free than were those of Luca Pacioli, Galileo and Leibniz. There may be good reasons that patronage nowadays is reflected not only in the dedications but also in the scholarly texts.[39] But then, at least, one might expect a minimal elaboration

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[38] A&V go on to claim that this approach "belongs to the prehistory of algebra," which, if at all, can probably best be explained as an acceptance of Knuth's understanding of the Babylonian texts *as he read them* as "algebra."

[39] However, Erich Kästner, 1965, p. 157, recommended that
Was auch immer geschieht:
nie dürft ihr so tief sinken,

of the concepts that are accepted, and here there is none. A&V, when adopting the notion of algorithmicality and in order to avoid ambiguity (p. 58), introduce a reasonable distinction between the paradigmatic example and the rule which it exemplifies. Unfortunately, the term "algorithm" is identified with the numerical example,[40] while the rule formulated without specified numerical values is a "procedure." Anybody who has ever learned modern programming to the level where "if" or "until" statements are introduced will know that an algorithm with branchings cannot be derived from a numerical example – at most from a plurality of numerical examples with commentaries explaining the choices made. But that is exactly what Heron (and so many others) avoid to present. *Metrica* I.5 explains how to find the height of an acute-angled triangle, and I.6 that of an obtuse-angled triangle. These stand in parallel, with no superior level leading to a choice. Introducing this superior level and the explicitly determinable choice is exactly what announces the beginning of algorithmic thought.[41]

A&V try to display their algorithmic idiom as a new insight of our own times. On p. 506 they claim not to be sure that "one knew how to compare texts of algorithmic character as such [...] in the epoch of Heiberg and Heath," as if comparison of calculational steps, numerical parameters etc. were not known. The problem, then as now, was and is to decide their pertinence for particular questions, as well as the statistical significance of observations; on that account, our choices, as those of our predecessors, may be disputable and even mistaken.

My next objection is that the reading of the volume is simply unpleasant. The tone is generally rude and arrogant, and the presentation often distorts what other workers have done – note 17 and the above characterization of Schöne's and Bruins' editions present modest examples. This rudeness is not only unpleasant (in which case it should perhaps not be discussed in a review) but also a way to entrap the reader. One of the more glaring examples is found on pp. 521-523.[42] On p. 521 Yves Guillaumin is quoted for this statement:

> It is clear that the *Podismus*, as also the extracts "from Epaphroditus and Vitruvius Rufus," function as kind of translation/adaptation of a Greek original which we must search for in what has been conserved of the Heronian tradition under the title *Geometrica*.

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von dem Kakao, durch den man euch zieht,
auch noch zu trinken!
But that was in 1932, during (the last year of) Weimar liberty.

40 Strictly speaking, the algorithm is "mis en oeuvre sur des exemples numériques," but since it has no existence outside this "being put to work" and is supposed distinct from the abstract set of prescribed steps, identification though packed in verbal cotton must be meant.

41 That is, the beginning is exactly in what was once called *algorism*, computation with Hindu-arabic numbers. Adding (for example) a number with digits *abc* to one with digits *def* introduces a first choice: *if* $c+f$ is smaller than 10 you do one thing, if 10 or more you do something different. And you go on moving toward the left *until* there are no more digits. Quite some work, of course, to formulate this in FORTRAN, but that is what genuine algorisms are about (as Knuth knew).

42 Please note that this has nothing to do with the question whether Guillaumin (and, presently, Heiberg) are right or wrong; it concerns the arguing style of the book under review.

Already on p. 517, n. 194, this has become in anticipation the "disputable assertion of Guillaumin, according to which the *Podismus* should be a translation of the *Geometrica*." The distortion is repeated on p. 522; and on p. 523, showing definitively their intellectual superiority, A&V list four criteria needed to recognize a translation-adaptation of a Greek *problem* (which is not what Guillaumin deals with, he speaks of texts in their entirety; "functions as kind of" / *fonctionne comme une sorte de* has completely disappeared). A&V dictate that:

– it deals with the same geometric object;
– the numerical data are the same;
– the question(s) is / are the same;
– the resolving procedure is the same.

That is more or less what the rest of us would call a translation simply, and definitely not what Guillaumin speaks about.

Heiberg is the victim of a sequence of similar distortions. On p. 498, in connection with the discussion of the possible "derivation" of the pseudo-Heronic writings from the *Metrica*, his popular exposition *Naturwissenschaften und Mathematik im klassischen Altertum* [43] is quoted for the opinion that the *Metrica* was "reshaped by the Byzantines, who left out the theory,[44] as elementary textbooks [*Rechenbücher*] and problem collections." From this A&V conclude that "derivation" (a term Heiberg does not use) "should thus have consisted in suppression of the demonstrations and conservation of the examples and the numerical procedures." This interpretation, reminiscent of Petrus Ramus's royal road to geometry, is then basis for a polemic that continues until p. 504.

On p. 517, the same popularization is quoted for the opinion that the *Metrica* "is oriented toward practice, namely that of surveyors, which since times immemorial was important in Egypt," and concerning the *Dioptra* that "one cannot avoid the assumption that the author was somehow connected to the education of surveyors." A&V reply with this scornful pearl:

> To think that the surveyors would justify the procedures by which they measured the terrain by giving themselves to the pleasures of geometric analysis of chains of givens [45] while using "a very complicated precision instrument," delicate and requiring a long preparation, that is really to express great optimism regarding the level of the demands and the formation of the practitioners of Antiquity.

Firstly, this disregards Herons's claim that the dioptra has "many and imperative" practical applications [46] – maybe Heron's claim is nothing but publicity, but

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43 HEIBERG, 1912b.

44 "Unter Weglassung der Theorie," not "durch." "Unter" refers to an accompanying circumstance, not the cause or means.

45 A pedantic aside: A&V forget that the chains are their reconstruction, following from their wish to connect the reasoning in the *Metrica* to the *Data*, and not in Heron's text.

46 Ed. SCHÖNE, 1903, p. 188.

that has to be argued; secondly, should we characterize the builders of the Parthenon (and all the other temples) as clumsy bunglers unable to perform precise measurements (A&V speak of "practitioners" in general)?[47] Finally, are A&V ignorant of the sophisticated level of mathematics taught in the early École polytechnique, far above what the students were later to use in their practice? Mathematics teachers of future practitioners often go beyond what the students will eventually need – after all, the practice with which the teacher is most familiar is *teaching*.[48]

On p. 534 (n. 239), I have the honour to come under (definitely milder) attack myself. As pointed out by A&V, the phrase *practica geometriae* was apparently first used by Hugh of Saint-Victor in the 1120s and a century later by Leonardo Fibonacci. The phrase means "the practice of geometry," an expression that hints at a specific epistemological stance: namely that *geometry* is a *scientia*, to which belongs a *practica*. A&V (mis)translate it as *géométrie pratique*, that is, practical geometry, and claim that the latter concept cannot be legitimately applied to other epochs and cultures (just after having said that it covers the same as Arabic *ʿilm al-misāḥa*, which is not completely true); it is also quite unclear, thus A&F, "what one should understand by it" (after which follow six lines of proposals).[49] It is therefore by "retroprojection" that I speak in a title about "Near Eastern Practical Geometry." Alexandre Vincent (who has the honour of an exclamation mark on his opinion) is similarly censured for having dared in 1858 to present an extract of the *Dioptra* as one of several specimens of *la géométrie pratique des grecs* (both of us are castigated on p. 534, n. 239). One wonders whether A&V are aware that Dominicus de Clavasio's *Practica geometriae* (ed. Busard 1965, pp. 535-537) deals with the quadrant, also a measuring instrument.

Exclamation marks are, in general, an important component of the rhetoric of the book. There are some 200 of them, mostly indicating that A&V find some opinion or statement funny / obviously mistaken / …, and that the reader is expected to take over that evaluation. The trick has the advantage that its user never needs to specify whether funny or scandalous or whatever, nor to explain why.

Sometimes, of course, attacks are justifiable though their use is not. P. 28 quotes B. L. van der Waerden for this (in English in the book under review):

> He [*scil*. Héron] also wrote […] a number of works on areas and volumes, the most popular of which is called *Metrics*. It is a very childish little book. Imagine: first 10 examples on the calculation of the area of a square, then 4 on the area of a rectangle, 14 on right triangles […]. Nothing but numerical examples, without proofs. Just like a cuneiform text.

47 The next page, on the other hand, first identifies Heiberg's *Feldmesser* with the French term *arpenteur*, which is unobjectionable; but next the *arpenteur* is believed to be, not the one who performs actual measurement but a specialist of the *cadastre*, a fiscal office.

48 I once taught physics to future building engineers; I had to explain to my students how the general Hilbert space theory they were taught could be transformed so as to serve (some of the physics teachers were no less ambitious on behalf of their discipline, of course).

49 May I suggest "geometry of practitioners;" what practitioners are they must be supposed to know, since they use the word with neither explanation nor reservation on p. 519.

As a [*sic*; correct in original/JH] example let us take the way in which the well-known "Heron's formula" for the area of an arbitrary triangle is explained: [*sic*/JH] And, after all, it is not very important. It is mankind's really great thoughts that are of importance, not their dilution in popularizations and in collections of problems with solutions. Let us rejoice in the masterworks of Archimedes and of Apollonius and not mourn the loss of numberless little arithmetic books after the manner of Heron.

As A&V justly observe on p. 29, one "needs only read, even cursorily, the beginning of the *Metrica* in order to question the description offered here." Strangely however, A&V do not reveal that van der Waerden is simply speaking about the *Geometrica* collection, which the words fit. A&V must have seen this – they point out that the account of the triangle formula [omitted here as well as in their quotation] comes from there, and not from the *Metrica*. Even here, A&V pick the most unkind interpretation one can find.

Amusing is that A&V make a *mistake of exactly the same kind themselves*. They refer the quotation to "Van der Waerden (1950, pp. 277-278)." As everybody familiar with the publishing history of *Science Awakening* knows, 1950 is the date of the first, *Dutch* edition.[50] The quotation instead is from the first edition of the English translation, dated 1954 (the bibliography has the same mistake).[51]

Precision of references is, generally speaking, not a strong point of the book. Stumbling on this English quotation from a Dutch book provoked me to make my first check of a reference. The next two were also less than precise: on p. 223, "Knorr, 1989, 507 n. 4" should refer to note 24; and on p. 54, n. 92, "Høyrup, 1997" should be "Høyrup, 1997a"[52] (the error is repeated on p. 55, n. 94). Then things went better, most of those references which I tried to follow (only that minority where I felt urged to inspect the source) were correct. However, on p. 493, n. 144, "Vitrac, 2005a, pp. 13-15" should point to p. 16.

More problematic than arrogance and occasional lack of precision are deliberate blind spots and prejudice. Since the mid-20th century it has been a common assumption that there are not only similarities but also links between Heronic and Pseudo-Heronic calculational geometry and Near Eastern, not least Babylonian mathematics (see quotations above from Neugebauer and Bruins; both underpin their opinions with precise text references). Some of the similarities may be explained as the outcome of similar tasks under the constraints of mathematics, but hardly all of them. In any case, A&V do not attempt any such explanation, instead they make an effort to avoid that the reader discover the possible non-Greek background to the Heronic and pseudo-Heronic kind of mathematics. In some cases there is no doubt that this is done quite deliberately. So, on p. 223, Heron states

[50] See, if documentation beyond library catalogues is needed, Soifer, 2015, pp. 404, 465 (but disregard the mistake on p. 235).

[51] In the first German edition from 1955, after a correction due to Bruins, the reference is corrected.

[52] Here Høyrup, 1997b.

(*Metrica* I.30) that a certain way (ascribed by him to "the ancients"/οἱ ἀρχαῖοι) to find the area of a circular segment seems to come from those who take the perimeter of the circle to be the triple of the diameter. In note 287 A&V then explain "as to the approximation $\pi = 3$, see Knorr, 507 note 4" (which, as mentioned above, should be "note 24"). The reader who is curious and stubborn enough will find that Wilbur Knorr simply mentions as a matter of course that "circle measurement via the constant 3" is "Babylonian." So, A&V avoid the dirty word by hiding it in a reference the reader can be supposed not to control or not to find. Worse, this six-line note in [Knorr 1989], discussing the same Heronian passage, points to the appearance of the same segment rule in Egyptian papyri from early Ptolemaic time, and to the mixture of Babylonian and Pharaonic methods which characterizes Demotic mathematics in general. It is unthinkable that A&V have not seen this, so if they did not know it already (hard to believe) they should have discovered that elucidation of Heron's project requires comparison with at least Demotic mathematics; not mentioning it looks like betrayal of the reader.

*Interdum dormit Homerus*. Had this been the only case, it might perhaps be an honest mistake. However, on p. 493 there is a remark about problem 3 of *Geometrica*, chapter 24 (a separate treatise, already itself a conglomerate of disparate origin).[53] This problem states a square area together with the perimeter to be 896 feet, and asks for separation of the area and the perimeter. A&V state that this is "one of the most discussed problems of the *Geometrica*." An appurtenant note (143) runs "See, for example, Vitrac, 2005a, pp. 13-15" (as mentioned, this should be "p. 16"). However, Vitrac does not "discuss" the problem, what one finds is a listing of the numerical steps and a parallel explanation of the calculation in symbols.[54] The reference thus only serves to disguise that other discussions of the problem type point to its appearance in the Old Babylonian mathematical corpus,[55] and show that it lives on until Luca Pacioli's *Summa* from 1494.

Page 493 also mentions problems in the same conglomerate that state the sum of circular diameter, perimeter and area and ask for their separation (also found, in different wording, in *Geometrica*/A+C). Here too, literature known to A&V would inform them and the reader that this problem is of Old Babylonian (or earlier) origin – and here too, the reference is to "Vitrac 2005a, pp. 10-13," which only sees a "possible manifestation of the perversity of mathematics teachers, known since long," or perhaps "an effect of the absolute primacy of the geometric figure in the whole of Greek mathematics, including that of algorithmic type."[56] On p. 454 it is suggested that the same problem type when found in *Geometrica*/AC may have been found by the compiler in "another book" – according

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[53] Høyrup, 1997a, p. 93.

[54] Vitrac, 2005, p. 16.

[55] One at least in found in an article A&V certainly know (Høyrup, 1997a), since they list it in their bibliography as (Høyrup, 1997b) and polemicize against it on p. 534, n. 239.

[56] Thus Vitrac, 2005, p. 16.

to the context by Heron. Even without knowing the Old Babylonian occurrence A&V should have seen that the inhomogeneous nature of the problem points to Babylonian or Demotic mathematics. However, they merely would "have liked to know which demonstrative justification" Heron would have given for it.

Many – not all – of these problematic features of the book grow out of and illustrate two fundamental tenets of A&V.

The first of these is revealed by the statement (p. 41) that "we know rigorously nothing" about possible sources of the *Metrica*, which is only true if "we" refers to A&V and to what they (accept to) know – cf. above on the Demotic calculation of the circular segment and on Heron's own reference to "the ancients" – and by a general refusal to mention any source that might illuminate how the *Metrica* and the metrological corpus were embedded in a Near Eastern tradition transcending the Greek world (and let us not forget that Hellenistic mathematics was also Near Eastern geographically). Beyond what was already mentioned it is remarkable that the *Liber mensurationum* is solely mentioned (p. 546) because it contains "Heron's formula" for the triangular area. A&V refer to Marc Moyon's forthcoming edition, and appear not to know about H.L.L. Busard's edition from [1968] (quite astonishing, it is used in publications they list). If they had looked into it they would obviously not have found any direct source for Heron – after all, the lost Arabic original is no earlier than the late eighth century CE, and perhaps to be dated several centuries later; but they would have discovered a puzzling parallel (namely the use of *semper*) to the use of καθόλου (and in the various constituents of the *Geometrica* καθολικῶς, ἀεί, παντός and πάντοτε) when a calculation makes use of numerical constants that do not depend on the given parameters. They would also have avoided repetition (on p. 546) of the fable that Fibonacci draws heavily on Savasorda's *Liber embadorum* for his *Practica geometrie* – a fable that originated when the *Liber mensurationum* was not known.[57]

Similarly, when P. Genev. gr. 259 is listed among the "so-called school papyri of geodesic contents," only select aspects of the terminology are mentioned, not its affinities with certain Seleucid and Demotic problems, nor with the *Liber mensurationum* (whence also Fibonacci) and the *Liber podismi*.

Various other deliberate oversights contribute to a picture of Greek mathematics, even of the Heronic and pseudo-Heronic type, as the outcome of culturally immaculate conception.

The other fundamental tenet is that Heron was a "scholar" (*un érudit*) and no mechanic or artisan (βάναυσος) – no doubt correct at a certain level, but stated on p. 26 as a premise which earlier workers have erred by missing, well before A&V present the arguments for their view. The difficulty with this conviction resides in the lack of differentiation when it comes to arguments around the two categories (in spite of a warning on p. 519 against too rigid distinctions concerning social and/or literary categories). If, for parallels, we look at Renaissance mathematics,

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57 CURTZE 1902, p. 5, if not before.

we find many kinds of "scholars" writing about practical mathematics. Pacioli and Tartaglia were both teachers of abbacus mathematics who worked themselves into the role of Euclidean scholars; Cardano was a physician and philosopher who took up abbacus mathematics, in part "from a higher vantage point," and who made vital discoveries; and Ramus was a literary scholar who wrote a little within mathematics without understanding much, and much Humanistic lore around the topic. Characterizing any of them merely as a "scholar" tells us very little. Similar, what do we learn about Heron and his work by putting him into a family which encompasses not only Archimedes but also Plutarch, Iamblichos and Macrobius – not to speak of the unknown author of "Aristotle's Mechanics," who paraphrases the *Metaphysics* by declaring that wonder may be excited by the works of art no less than by that which happens in accordance with nature?

So, one might have wanted A&V to look closer at what goes on in the *Metrica*; that would have forced them to discover conspicuous fault lines within the work which cannot be explained away by assumed alterations of the text, and which tell us something about Heron's approach.

The very first proposition (p. 150) runs "let there be an oblong area ΑΒΓΔ having ΑΒ of 5 units and ΑΓ of 3 units." Then Heron states that a right-angled parallelogram is said to be contained by the two segments containing a right angle, and that the area contained by ΑΒ and ΓΔ is right-angled; *therefore* the area will be 15 units. As proof he suggests to divide the sides into 5 respectively 3 parts and to draw parallel lines through the dividing points. The area is then declared to consist of 15 unit areas, with no intervening argument. Nothing is said about multiplication (although a link to Euclid's definition of that operation would be easily made on the basis of the diagram), nor about what to do in case the sides are not integer. Apart from the (indeed missing) reference to Euclid's terminology, Socrates would easily have guided Menon's slave boy to construct this "proof." Later – namely when the term μέθοδος has given way to "synthesis" – we find genuine proofs in Euclidean style. It would seem worthwhile at least to investigate whether Heron was rewriting an "illiberal" handbook for practical action in more scholarly style (more or less as Cardano was to do in later times); even though we have no information about such books dealing with practical geometry in Greek before Heron's times, we know from Aristotle that they existed in other fields (*Politics* $1258^{b}39$–$1259^{a}2$, mentioning agriculture and fruitfarming as examples among others).

Even after μέθοδος has given way to synthesis, there are indications that the text consists of several layers. This can be seen in the sequences I.5–9 and I.27–33. Since I have discussed these in [Høyrup, 1997b], and since the argument in complex, I shall abstain from details here, noticing only that A&V distance themselves on p. 55 from my observations that the presentation of "Heron's formula" in I.8 (with a theoretical lemma in I.7) is an interpolation borrowed from *Dioptra* 30 interrupting the coherent flow of the text between I.6 and I.9 (no doubt made by Heron himself but still, textually seen, an interpolation); on p. 56, however, they change their mind, suggesting Heron to have borrowed from an already finished redaction of the *Dioptra*, and that the context of the *Metrica* asked for the insertion of the theoretical lemma, which had not been needed in the *Dioptra* (exactly as I had explained).

Finally, there are a some puzzling mistakes that cannot be explained from some *parti pris*; I shall mention only two examples. On p. 195, n. 189 (referring to *Metrica* I.18), it is observed quite correctly that a calculation corresponds to the approximation $\sqrt{5} \approx 2^1/_4$. Then it is stated that "in a perspective where approximations to square roots are necessarily linked to side- and diagonal-numbers, this choice is inexplicable." What is inexplicable is A&V's statement: side- and diagonal numbers, indeed, only serve (and can only serve) to find approximations to $\sqrt{2}$. $2^1/_4$, on its part, is the value that follows from the algorithm explained in *Metrica* I.8.

On p. 54, as one of the arguments that *Metrica* I.4 is inauthentic [58] it is stated that a circularity in the argument would not have escaped Heron. Actually, there is no circularity, only a tacit assumption that an acute-angled triangle has internal heights only, whereas an obtuse-angled triangle has external heights. Then follows (and that is what is seen by A&V as circular) inversions of simplified versions of *Elements* II.12–13,[59]

In obtuse/acute angled triangles the square on the side subtending the obtuse/acute angle is greater/smaller than the squares containing the obtuse/acute angle.

Heron first summarizes the inversion, and then gives the proof for the case of "smaller:" namely that if the angle were right, the sum would be equal, and if obtuse, greater. This double *reductio ad absurdum* is certainly both trivial and pedantic, and no practitioner would have felt the need for it; but it is not circular, and if anything, it proves the author of the passage (whether Heron or somebody interfering with his text) to be a scholar rooted in the silver age of commentaries and pedagogical redactions – the age of "Deuteronomic texts," in Reviel Netz' words.[60]

All in all: A&V have produced a valuable edition, and anybody working on Heron's *Metrica* will be grateful to them.[61] When it comes to their further considerations, be it concerning the nature and conditions of Heron's project, be it concerning the metrological corpus, their very selective choice of and use of available evidence calls for caution; for the same reason I have abstained from reporting these opinions as if they had been results. A&F should be listened to attentively but also critically.

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[58] The others arguments, of a philological nature, are not necessarily more pertinent; but they are not mistaken in themselves.

[59] I follow (and curtail) the translation in Heath, 1926, I, pp. 403, 440.

[60] Netz, 1998.

[61] However, Bruins' facsimile should still be kept at hand. Cf. note 8.

Additional note, July 2018

Three months after delivering the final manuscript I have got access to the article in which Giancarlo Prato identifies the copyist of **S** as Ephrem (*Due postille paleografico-codicologiche*, pp. 279-291, in F. Berger *et al.* (eds.), *Symbolae berolinenses für Dieter Harlfinger*, Amsterdam, Hakkert, 1993). Prato goes no further than stating this as his "opinion" ("a mio avviso"), basing himself on the style of the handwriting; at the same time he observes deviations from the habits of Ephrem as known from other copies from his hand. I shall leave undecided whether this can justly be counted as irrefutable evidence in spite of Prato's own more cautious words.

Prato turns out to share Schöne's and Bruins's opinion about the copyist's lack of mathematical competence, and quotes the same striking example as Bruins.

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